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# Lensing by Gravitational Waves from Supernovae

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Gravitational waves can act as gravitational lenses and create multiple images of a light source. This situation is much more interesting than single-image lensing because of the associated high-amplification events that may lead to the indirect detection of gravitational waves. It is proposed to observe the effect due to gravitational waves generated in supernova explosions.

The importance of gravitational wave detection for modern physics can hardly be overemphasized. While the direct detection of gravitational waves is a major undertaking of experimental gravitational physics, it is worthwhile to consider the possibility of an indirect detection of gravitational waves via the effects that they induce when they cross a light beam in extraterrestrial space. Many authors have studied the redshift perturbations, deflections, and scintillation induced by gravitational waves on a light beam, particularly for cosmological gravitational waves. It was soon discovered that, when there is a single image of the light source, the amplification effect of a light beam propagating through gravitational waves is of second order in the wave amplitude, and therefore negligibly small [1]. This result was obtained by using the Raychaudhuri equation for a congruence of null rays in the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1.1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ ($|h_{\mu\nu}| \ll 1$) describe gravitational waves. This conclusion is valid also in scalar–tensor theories of gravity [2] and was obtained by using the same method of ref. 1. The Raychaudhuri equation, however, is not able to describe situations in which a gravitational lens splits the light beam and creates multiple images of a distant light

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source, as is well known from the theory of gravitational lensing by *ordinary* gravitational lenses (i.e., mass concentrations) [3]. The hypothetical possibility that this situation occurs when the lens is a gravitational wave was conceived of in refs. 4 and 5, but it was not proved that this strong lensing regime is possible in real life, for realistic gravitational waves. Multiple imaging, if possible, would be observationally much more interesting than the single-image situation, and could potentially lead to the indirect detection of gravitational waves. In fact, if multiple images are possible at all, the associated high-amplification events that necessarily occur when a light source crosses a caustic generate sudden bursts of light that have a much higher chance of being detected than a tiny modulation of the light intensity or change of apparent position of a light source. This situation is well known in the case of lensing by ordinary mass distributions, and the discovery of such lens systems has made gravitational lensing a very active area of astronomy in the last 20 years.

To overcome the limitations of the Raychaudhuri equation (which is restricted to the case in which the null rays form a congruence and do not intersect each other) in the case of multiple imaging, one would like to use the vector formalism of ordinary gravitational lensing [3]. The first difficulty one has to face consists in the fact that this formalism is based on Fermat's principle, and therefore it is valid only in conformally stationary spacetimes. Fortunately, a recent and less-known generalization of Fermat's principle [6] applies also to nonstationary spacetimes. Using this generalized formulation, one proves that the vector formalism of ordinary gravitational lensing also describes lensing by a gravitational wave (a highly nonstationary spacetime), provided that a suitable expression for the deflection angle is given [7]. Naively, everything works as if the vector formalism holds with the deflection angle $\delta p^\mu$ computed from the geodesic equation. We refer the reader to refs. 7 and 8 for this calculation. Let

$$p^\mu = p^\mu_{(0)} + \delta p^\mu = (1,\ 0,\ 0,\ 1) + \delta p^\mu \tag{1.2}$$

be the tangent 4-vector to the spacetime trajectory of a photon whose unperturbed 3-dimensional path is parallel to the $z$ axis. One finds

$$\delta p^\mu = \frac{1}{2}\int_S^O dz\ (h_{00} + 2h_{03} + h_{33})^{,\mu} + O(h^2) \tag{1.3}$$

the integral being computed along the unperturbed photon path. Gravitational lensing is then described, as usual, by a plane-to-plane mapping from the lens plane (orthogonal to the optical axis, and passing through the position of the lensing wave) to the source plane (similarly passing through the position of the light source). Due to transversality, the action of the gravitational wave can be reduced to that of the "projection" of the wave propagating in the direction orthogonal to the unperturbed photon path [8]. The geometrical mapping is described by the lens equation

$$s^A = x^A + \frac{D_L D_{LS}}{D_S}\, \delta p^A(\mathbf{x}) \tag{1.4}$$

where $A = x, y$ and $D_L$, $D_{LS}$, and $D_S$ are, respectively, the observer–lens, lens–source and observer–source angular diameter distances. Multiple images occur if and only if the geometrical map fails to be invertible, i.e., the Jacobian determinant $J$ of the transformation vanishes. One has

$$J = 1 + J_1 + J_2 \tag{1.5}$$

where

$$J_1 = \sqrt{f(\alpha)}\, D_S \frac{\partial(\delta p^A)}{\partial x^A} \tag{1.6}$$

$$J_2 = f(\alpha)\, D_S^2 \left[\partial_x(\delta p^x)\, \partial_y(\delta p^y) - \partial_y(\delta p^x)\, \partial_x(\delta p^y)\right] \tag{1.7}$$

$$\alpha = D_{LS}/D_S \tag{1.8}$$

$$f(\alpha) = \alpha^2(1 - \alpha)^2 \tag{1.9}$$

In order of magnitude,

$$J_1 \simeq Dh^2 / P \ll J_2 \simeq (Dh/P)^2 \tag{1.10}$$

when $D \gg P$, where $P$ is the gravitational wave period. For, say, $f(\alpha)$ in the range 1/100–1/16 around its maximum, the condition for the creation of multiple images $J = 0$ corresponds to $J_2 \simeq O(1)$ [since $J_2$, given by Eq. (1.7), is an oscillating quantity]. A straightforward order-of-magnitude estimate gives the approximate condition for multiple imaging by gravitational waves:

$$\frac{h}{P} \geq 4 - 10 \frac{c}{D} \tag{1.11}$$

This condition on the "strength" $h$ and the size $P$ of the lens is analogous to the well-known approximate condition for multiple imaging by ordinary mass distributions, $\Sigma \geq c^2/(4\pi G D)$, where the surface density $\Sigma$ of the lens measures the "strength" of the lens in conjunction with its size.

Is the condition for multiple imaging (1.11) satisfied for realistic astrophysical systems? Order-of-magnitude estimates for the gravitational wave amplitude [8] provide the answer:

- Stellar core collapse: there is a relevant volume of the space of parameters ($D$, the efficiency of the gravitational wave generation process, and the impact parameter of light rays) for which the condition (1.11) is satisfied.

- Final decay of a neutron star–neutron star binary: condition (1.11) is satisfied in a large range of values of the parameter space.
- Black hole collisions: (1.11) is satisfied for many geometries.
- The binary pulsar PSR 1913 + 16 emits gravitational waves too weakly to generate multiple imaging or to create an appreciable amplification of a light beam. This conclusion applies, of course, to all binary systems that radiate gravitational waves more weakly than PSR 1913 + 16. In particular, this conclusion defeats the ongoing attempts to detect scintillation due to gravitational waves from the binary CM Del using VLBI observations of the radio source 2022 + 171 [9].[3] Unfortunately, the same conclusion applies to the observations proposed for other sources [5, 10].
- The gravitational wave background, on average, does not create multiple images.

It is thus demonstrated that gravitational waves generated in catastrophic events are able to split the image of a light source thanks to the balance between small values of $h_{\mu\nu}$ and large values of $D$ in Eq. (1.7). Unfortunately, these events are generally rare and short-lived, and one cannot predict in advance their location in the sky in order to point a telescope there. An exception is the case of supernova explosions: because of their intrinsic optical brightness, supernovae can be located unambiguously, and one can observe a luminous object A at a direction $\theta$ in the sky from the supernova where the emitted gravitational waves are known to be at the time $\tau$ after the supernova explosion. If $d$ is the distance to the supernova, the gravitational wave pulse has traveled a distance $d \sin\theta$ orthogonal to the line of sight to A, the lensing event takes place, and the lensed light reaches the observer at the time $d(\sin\theta + \cos\theta) \simeq d(1 + \theta) > d$. If a luminous object A is located at the angle $\theta$ with the direction to the supernova and at about twice the distance $d$ from us [in order to maximize the factor $f(\alpha)$ in Eq. (1.7)], one predicts luminosity variations of A, and its splitting into multiple images at a time $d(1 + \theta)/c$. The flux variations can be detectable even if the angular separation between the multiple images is too small for them to be resolved. Note that this situation occurs, e.g., in the well-known microlensing by stars; for a solar mass microlens, the separation between the multiple images is of order $10^{-6}$ arcsec, but the flux variations are clearly observable. For sufficiently small $\theta$, the distance from the observer to the space point where the

[3] In addition, the relative distances between observer, lens, and source for this system are very unfavorable, yielding a very small factor $f(\alpha) = 2 \cdot 10^{-7}$.

lensing event takes place is small enough that the gravitational wave can still generate multiple images, and the time $d(1 + \theta)/c$ allows an observation within a short time from the supernova explosion.

To show that the high-amplification effect is possible, consider the following example: gravitational waves generated in a stellar core collapse have a magnitude that can be estimated, e.g., using standard results on collapsing homogeneous ellipsoids [11]. If the collapsing core keeps bouncing, the gravitational wave amplitude at the edge of the wave zone ($r > 3 \cdot 10^7$ cm) is $h_S \simeq 10^{-2}$ [11]. By requiring that a high-amplification event occurs at a time $\sim$100 sec after the supernova explosion, one obtains an impact parameter $r \simeq 3 \cdot 10^{12}$ cm for the light rays. The high-amplification event occurs if $d \sim 6 \cdot 10^{16}$ cm; the angular separation between the lensed object and the supernova is then $\theta \sim r/d \sim 10$ arcsec. High amplifications due to gravitational waves are actually possible in realistic astrophysical situations relevant for observations. One potential obstacle to the observability of the effect is the smearing of the brighter and dimmer areas of the images if the size of the lens (the gravitational wave) is smaller than the size of the light source.[4]

The details of the lensing phenomenon, including the shape of the images, their flux, and time variability, will be presented in a forthcoming publication.

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